Excited States with Time-Dependent Density Functional Theory and with Linear Vibronic Coupling Models

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July 8th, 2022

"Excited States and Nonadiabatic Dynamics CyberTraining Workshop 2022" in Buffalo, NY



Time-dependent density functional theory

What are people using to compute excited states?

Time-dependent density functional theory: Foundations

Regular DFT only works for an non-degenerate ground state, based on the scope of the Hohenberg-Kohn theorems.

Huix-Rotllant, Ferré, Barbatti: "Time-Dependent Density Functional Theory" in "Quantum Chemistry and Dynamics of Excited States" by González, Lindh (2021).

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Theoretical foundation of TDDFT:

- ▶ 1. Runge–Gross theorem: Correspondence between TD electron density $\rho(\vec{r}, t)$ and TD external potential.
- 2. Runge-Gross theorem: Least action principle, giving hints how the TD electron density should evolve.

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Thus, TDDFT is formally possible. It requires a TD functional, which has some additional requirements versus the Hohenberg–Kohn functional. E.g.,

- Causality (between external perturbation and density)
- Time non-locality (memory effect)
- Spatial non-locality (formally, local density approximation is incorrect in TDDFT)

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Time-dependent density functional theory: Kohn-Sham

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TD density taken as density of TD KS determinant:

$$\rho(\vec{r}, t) = \sum_{i} f_{i} |\phi_{i}(\vec{r}, t)|^{2}$$
(1)

Time evolution according to TD-KS equations:

$$\hbar \frac{\partial}{\partial t} \phi_i(\vec{r}, t) = \hat{h}^{\text{KS}}[\rho] \phi_i(\vec{r}, t)$$
(2)

Here, $\hat{h}^{\text{KS}}[\rho]$ should depend on $\rho(t') \forall t' \leq t$. In practice, it usually only depends on $\rho(t)$, called the **adiabatic local density approximation (ALDA)**.

Time-dependent density functional theory: Solving the TD Kohn-Sham equations

The first way to solve the TD-KS problem is real-time TDDFT (RT-TDDFT):

- 1 Start with the initial electron density
- **2** Introduce a periodic oscillating perturbating electric field $\vec{\mathcal{E}}(t) = \vec{\mathcal{E}}_0 \cos(\omega t)$
- 3 Simulate the temporal evolution of the electron density numerically
- Obtain spectroscopic information from the Fourier transformation of the dipole moment autocorrelation function.



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Very flexible (for weak and strong fields, good scaling with system size) but generally demanding.

Time-dependent density functional theory: Solving the TD Kohn-Sham equations

The second way to solve the TD-KS problem is linear-response TDDFT (LR-TDDFT):

- For weak external perturbations, use response theory in frequency domain
- 2 Derivation: Start at linear density $\delta \rho(t)$, Fourier transform to frequency domain, derive response function, solve for poles of response function
- **3** This leads to the Casida LR-TDDFT equation:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{B} & -\mathbf{A} \end{pmatrix} \begin{pmatrix} \vec{X} \\ \vec{Y} \end{pmatrix} = \omega \begin{pmatrix} \vec{X} \\ \vec{Y} \end{pmatrix}$$
(3)

where matrices A and B have matrix elements:

$$A_{ia,jb} = \delta_{ij}\delta_{ab}(\epsilon_a - \epsilon_i) + \iint \phi_i^*(r)\phi_a(r) \left[\frac{1}{|r - r'|} + \hat{f}_{xc}\right]\phi_b^*(r')\phi_j(r')drdr'$$
(4)

$$B_{ia,jb} = \iint \phi_i^*(r)\phi_a(r) \left[\frac{1}{|r-r'|} + \hat{f}_{xc} \right] \phi_j^*(r')\phi_b(r') dr dr'$$
(5)

Time-dependent density functional theory: LR-TDDFT

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{B} & -\mathbf{A} \end{pmatrix} \begin{pmatrix} \vec{X} \\ \vec{Y} \end{pmatrix} = \omega \begin{pmatrix} \vec{X} \\ \vec{Y} \end{pmatrix}$$
(6)

Some notes:

- A and B are formally depending on ω, but not when using the ALDA. This approximation is present in most TDDFT implementations.
- The equation has many eigenvalues ω that are the excitation energies of the system.
- For each ω , there will be a pair of \vec{X} and \vec{Y} vectors that describe the response of the density:

$$\delta\rho(r) = \sum_{ia} X_{ia}\phi_a^*(r)\phi_i(r) + Y_{ia}\phi_a(r)\phi_i^*(r)$$
⁽⁷⁾

- Here, the vector \vec{X} describes excitations $i \rightarrow a$ and \vec{Y} describes deexcitations $i \leftarrow a$.
- Often used is the Tamm-Dancoff approximation (TDA) that sets **B** = 0 and thus $\vec{Y} = 0$:

$$\mathbf{A}\vec{X} = \omega\vec{X}.$$
 (8)

Time-dependent density functional theory: Connection to HF and CIS



TDDFT: How good is it? General points

These points are valid for LR-TDDFT with the ALDA (most TDDFT calculations):

- > TDDFT naturally depends on the reference state, hence on the used XC functional
- Errors in the excitation energy are usually up to ± 0.5 eV, in good cases ± 0.2 eV
- Pure functionals tend to give too low excitation energies
- More sophisticated functionals tend to give better results: LDA < GGA < Hybrid < Range-separated hybrid < Double hybrid</p>

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The accuracy is also strongly dependent on the type of excitation:

- Local excitations ($n\pi^*$, $\pi\pi^*$, metal centered, ...): Often good with hybrids.
- Rydberg excitions: Often rather bad with errors >> 0.5 eV (due to self-interaction error and too low ionization potentials).
- Charge transfer excitations (e.g., push-pull compounds or metal complexes): Requires range-separated hybrids, otherwise appear much too low and spurious ghost states appear. Can be identified by having little hole-electron overlap.
- Core excitations (for X-ray spectra): Similar to CT excitations, can be handled with empirical shifts, or with range-separated hybrids.
- Higher excitations: Can be very wrong, due to the neglect of doubly/higher excited determinants and the too low ionization potential of DFT.

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TDDFT: How good is it? Dependence on reference state

- If DFT does not work, then TDDFT does not work
- Degenerate ground states: open-shell metal complexes, ...
- Degenerate ground states are also encountered in many dissociations

Problems in dissociation (attributed to KS-DFT, not TDDFT):

- RKS keeps electrons in pairs, precluding homolytic dissociation
- UKS allows unpaired electrons, but is spin-contaminated (very heavily in dissociation)

A conical intersection is a region where two PESs touch each other. They have a special, important topology:

$$\mathbf{H}^{\text{diab}}(x,y) = \begin{pmatrix} H_{11}(x,y) & H_{12}(x,y) \\ H_{12}(x,y) & H_{22}(x,y) \end{pmatrix} = \begin{pmatrix} x & y \\ y & -x \end{pmatrix}, \quad \mathbf{H}^{\text{adia}}(x,y) = \begin{pmatrix} -\sqrt{x^2 + y^2} & \frac{1}{\sqrt{x^2 + y^2}} \\ & \sqrt{x^2 + y^2} \end{pmatrix}.$$
 (9)

Conical intersection forms when two conditions hold along two different DOFs:

- $H_{11} = H_{22}$: here at x = 0
- ► *H*₁₂ = 0: here at *y* = 0

In TDDFT there are two problems:

- If $H_{11} = H_{22}$, then KS-DFT is not well-defined and can produce instabilities.
- In TDDFT, the ground state is artificially fixed to one KS determinant and never mixes with the excited states. Hence, there is no coupling and $H_{12} = 0$ everywhere (not only at y = 0).

TDDFT: How good is it? Conical intersections II



TDDFT for nonadiabatic dynamics:

- Is one of the cheapest, easiest to use, most available methods
- Produces reasonable PESs for many cases
- Avoid transitions between TDDFT states and reference state (general caveat for single-reference methods)
- > Pay attention to the system and kinds of states, choose functional wisely
- Avoid dissociation or extreme deformation of the molecule

Example of what works well:

• Dynamics that start in S_n , relaxes to S_1 , and then undergoes ISC to T_n and later to T_1

TDDFT: Computational cost of nonadiabatic dynamics



What can we do when ab initio methods are too expensive?



Linear vibronic coupling (LVC)

Use identical harmonic oscillators,

Coordinate Q

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F. Plasser, S. Gómez, M. Menger, S. Mai, L. González, PCCP 21, 57, (2019).

Linear vibronic coupling (LVC)

- Use identical harmonic oscillators,
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- and add linear vibronic couplings



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Linear vibronic coupling (LVC)

- Use identical harmonic oscillators,
- shift to get diabatic state,
- repeat,
- and add linear vibronic couplings
- ... to get adiabatic states!
- Can also include spin-orbit couplings.
- + Speed up: 3-5 orders of magnitude vs. ab initio
- Only for rigid molecules and short time scales

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Linear vibronic coupling model: Multidimensional models



Superscripts: indices of diabatic states, subscripts: normal modes.

Linear vibronic coupling model: Multidimensional models



Superscripts: indices of diabatic states, subscripts: normal modes. Definition of normal mode coordinates:

$$Q_i = \sqrt{\omega_i} \sum_{a}^{\text{atoms}} \sum_{d}^{3} K_{ad,i} \sqrt{M_a} (r_{ad} - r_{ad}^{\text{ref}})$$
(10)

Linear vibronic coupling model: Parameters

Set of parameters for harmonic oscillators:

- ▶ Normal mode frequencies ω_i : $3N_{\text{atom}}$ 6 parameters
- Normal mode transformation matrix with elements $K_{ad,i}$: $(3N_{atom} 6)(3N_{atom})$ parameters
- Normal mode reference geometry with coordinates r_{ad}^{ref} : $3N_{\text{atom}}$ parameters
- ▶ Atom masses *M_a*: *N*_{atom} parameters

In SHARC, these parameters are contained in the file V0.txt.

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Set of parameters for excited states:

- Vertical shifts ε^{α} : N_{states} parameters
- Gradient terms κ_i^{α} : $(3N_{\text{atom}} 6)N_{\text{states}}$ parameters
- Linear vibronic couplings $\lambda_i^{\alpha\beta}$: $(3N_{\text{atom}} 6)N_{\text{states}}^2$ parameters
- Constant spin-orbit couplings $\eta^{\alpha\beta}$: $N_{SO \text{ states}}^2$ parameters
- Dipole moment terms $\mu_d^{\alpha\beta}$: $3N_{\text{states}}^2$

In SHARC, these parameters are contained in the file LVC.template.

Linear vibronic coupling model: Obtaining parameters

Parameters for the harmonic oscillators:

- ① Choose desired reference state (often S₀, PES should look similar to relevant states)
- 2 Optimize minimum of reference state
- 3 Do frequency calculation of reference state

The SHARC script wigner.py can then produce the file V0.txt.

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Parameters for excited states:

- **1** Run a single point calculation at reference geometry:
 - Excitation energies: ε^{α}
 - (Transition) dipole moments: $\mu_d^{\alpha\beta}$
 - Spin–orbit couplings: $\eta^{\alpha\beta}$
 - Gradients transformed into normal mode coordinates: κ_i^{α}
- 2 Run 6Natom displaced single point calculations
 - Compute numerical derivatives of diabatic Hamiltonian: $\lambda_i^{\alpha\beta}$

This involves three steps, (i) using setup_LVCparam.py, (ii) running calculations in all created folders, (iii) using create_LVCparam.py to extract parameters and write the file LVC.template.

Linear vibronic coupling model: Acetone

Complex LVC model

- Model for n3p Rydberg and ππ^{*} states (grey box)
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- 24 ω_i
- ▶ 720 K_{ad,i}
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- 49 ε^α
- 392 nonzero κ_i^{α}
- 7280 nonzero $\lambda_i^{\alpha\beta}$

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LVC: Computational cost of nonadiabatic dynamics



Summary

Time-dependent density functional theory (TDDFT)

- A lot of formal math and derivations
- Practical LR-TDDFT with ALDA works like CIS, but is more accurate
- Cheap and easy to use, but errors depend strongly on system, state, and functional
- Avoid conical intersections to the reference state and dissociations in nonadiabatic dynamics

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Linear vibronic coupling (LVC) models

- Analytical model for coupled PESs
- Shifted and coupled multi-dimensional harmonic oscillators in normal mode coordinates
- Works well for rigid molecules with similar states and extremely easy to parametrize
- Extremely efficient (3-5 orders of magnitude faster than ab initio)

Thank you for your attention!

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My further thanks goes to:



