



TDDFT

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Casida Formulation

Perturbed density \rightarrow first-order correction

Linear response approach \rightarrow frequency domain

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$1 = (X|X) - (Y|Y)$$

Cannot be used to describe excitations in intense fields

$$A_{ia,jb} = \delta_{ij} \delta_{ab} (\varepsilon_a - \varepsilon_i) + (ia|F_H + F_{xc}|jb)$$

$$B_{ia,jb} = (ia|F_H + F_{xc}|jb)$$

$$F_{xc}(r_1, r_2) = \frac{\partial^2 f}{\partial \rho(r_1) \partial \rho(r_2)}$$

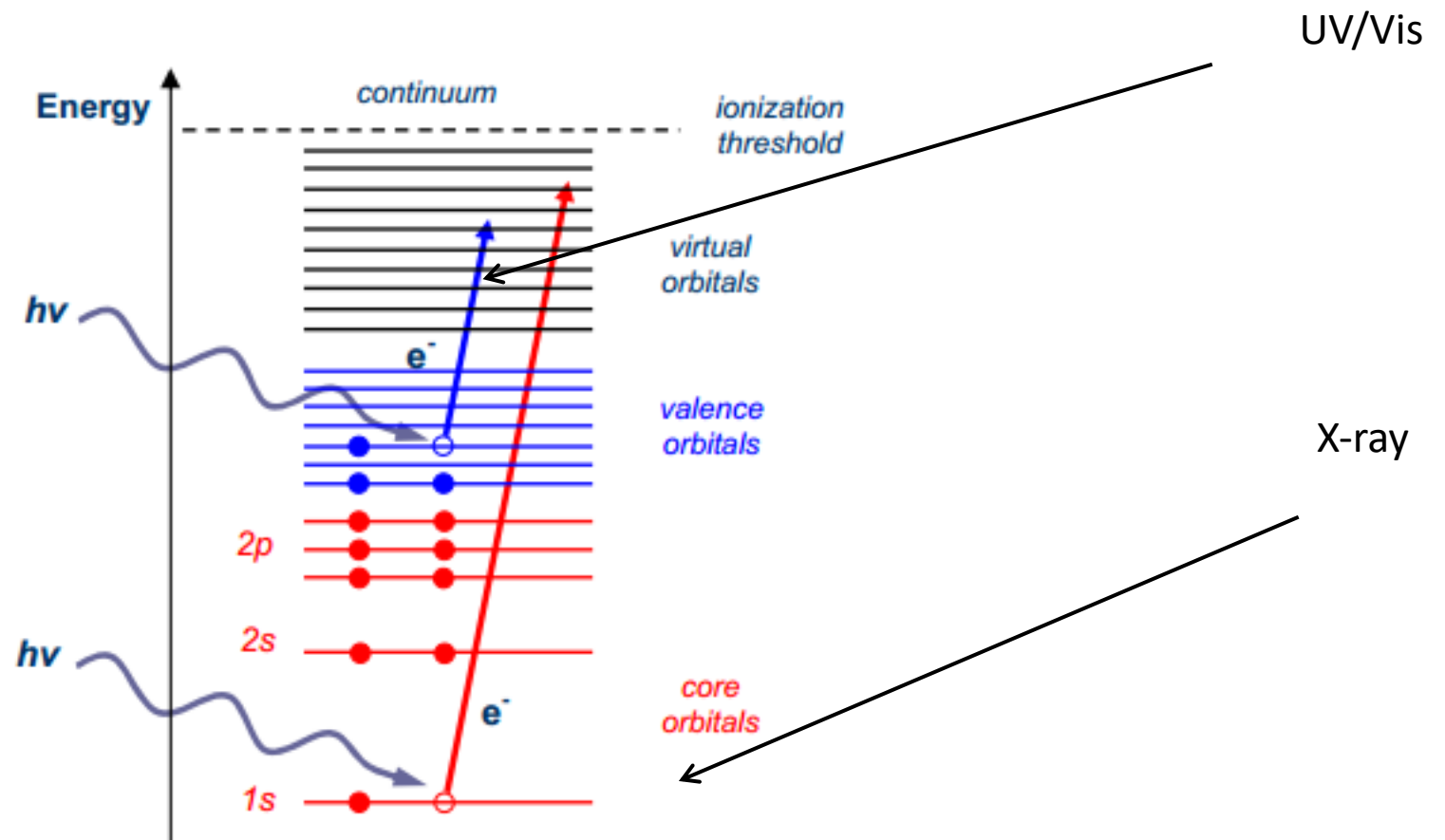
Time-Dependent Density Functional Theory,
Marques et al, Springer 2006

Time-
Dependent
DFT

Excited State Calculations with TDDFT

- Optical properties (UV/Vis)
- Pre- and near-edge X-ray absorption (XANES)
- TDDFT Gradients/Optimization

UV/Vis & X-ray Spectroscopy



Basic UV/Vis Input

```
geometry
O      0.00000000    0.00000000    0.12982363
H      0.75933475    0.00000000   -0.46621158
H     -0.75933475    0.00000000   -0.46621158
end
```

```
basis
  O library 6-31G**
  H library 6-31G**
end
```

```
dft
  xc b3lyp
end
```

```
tddft
  nroots 10
end
```

```
task tddft energy
```

Sample UV/Vis Output

Root	1	singlet	b2		0.294221368 a.u.	8.0062 eV

Transition Moments	X	-0.00000	Y	0.26890	Z	0.00000
Transition Moments	XX	-0.00000	XY	0.00000	XZ	0.00000
Transition Moments	YY	-0.00000	YZ	-0.08066	ZZ	-0.00000
Dipole Oscillator Strength						0.0141833591
Electric Quadrupole						0.0000000009
Magnetic Dipole						0.0000001219
Total Oscillator Strength						0.0141834819

Occ.	5	b2	---	Virt.	6 a1	1.00002 X

Root	2	singlet	a2		0.369097480 a.u.	10.0437 eV

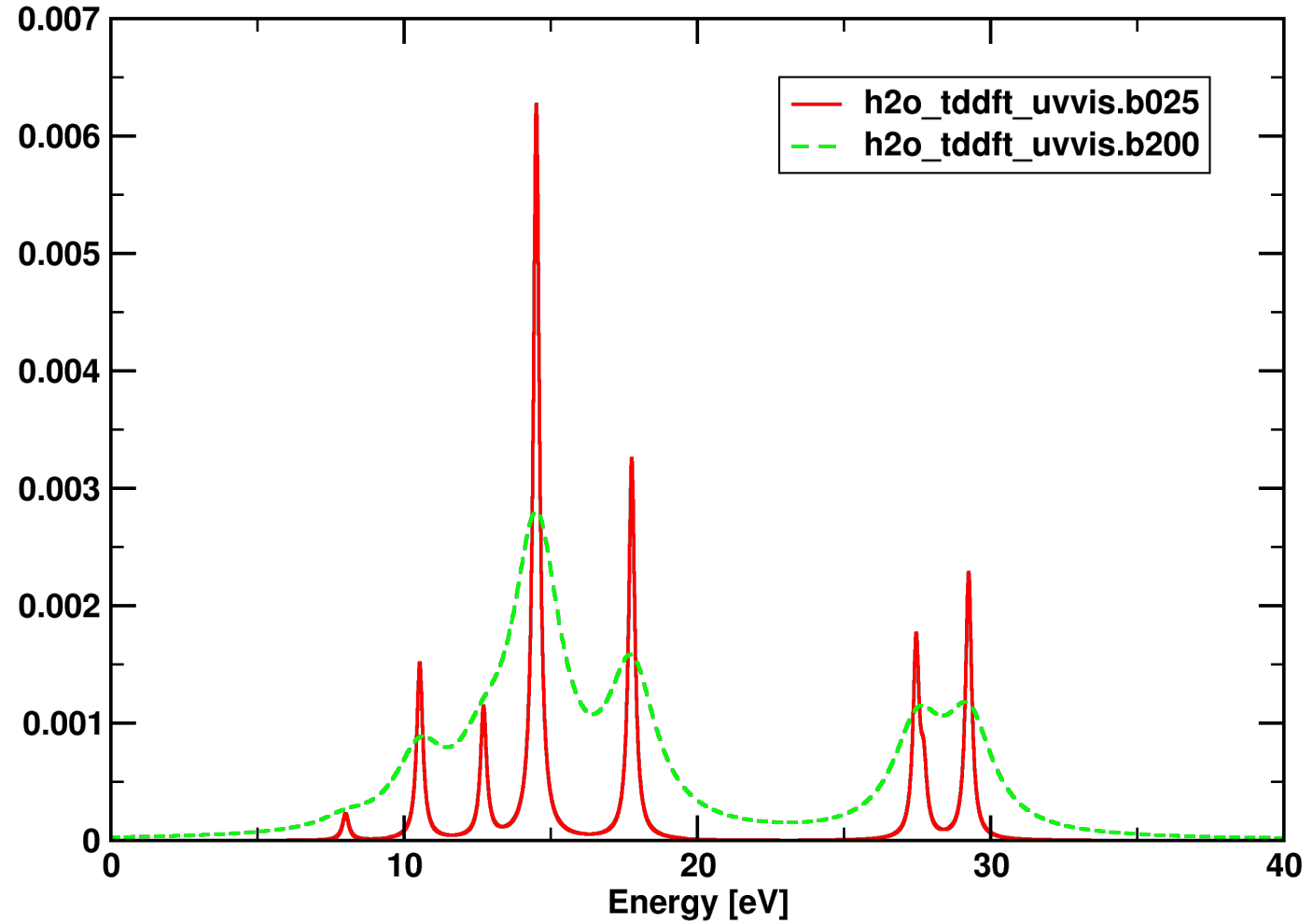
Transition Moments	X	-0.00000	Y	0.00000	Z	-0.00000
Transition Moments	XX	-0.00000	XY	0.24936	XZ	0.00000
Transition Moments	YY	0.00000	YZ	-0.00000	ZZ	-0.00000
Dipole Oscillator Strength						0.0000000000
Electric Quadrupole						0.0000000166
Magnetic Dipole						0.0000003348
Total Oscillator Strength						0.0000003515

Occ.	5	b2	---	Virt.	7 b1	-0.99936 X

Root	3	singlet	a1		0.387064731 a.u.	10.5326 eV

Transition Moments	X	-0.00000	Y	-0.00000	Z	0.60463
Transition Moments	XX	-0.62351	XY	0.00000	XZ	0.00000

UV/Vis Spectrum (different broadenings)



```
python3 nw_spectrum.py -b0.25 -p5000 < h2o_tddft_uvvis.out>h2o_tddft_uvvis.data.b025  
python3 nw_spectrum.py -b2.00 -p5000 < h2o_tddft_uvvis.out>h2o_tddft_uvvis.data.b200
```

Basic XANES Input

```
geometry units angstrom noautosym nocenter
  O 0      0      0
  C 0      0.00    1.15
end
basis
  * library aug-cc-pvtz
end
dft
  xc b3lyp
end
task dft optimize
dft
  xc BeckeHandH
end
tddft
  cis
  ecut -15 ←
  nroots 30
  notriplet
  thresh 1e-04
end
task tddft
```

Defines the initial state(s)
All (core) states below energy of -15 au
are taken as initial states. No restriction
on final states

Alternate input for XANES

```
geometry units angstrom noautosym nocenter
O 0      0      0
C 0      0.00    1.15
end
basis
  * library aug-cc-pvtz
end
dft
  xc b3lyp
end
task dft optimize
dft
  xc BeckeHandH
end
tddft
  cis
  ewin -20.0 -10.0 ←
  nroots 30
  notriplet
  thresh 1e-04
end
task tddft
```

Defines the initial state(s)
with energy windows

Sample XANES Output

Root 1 singlet a 19.467775490 a.u. 529.7454 eV

Transition Moments X -0.05038 Y 0.00001 Z 0.00000
Transition Moments XX 0.00000 XY -0.00000 XZ 0.00222
Transition Moments YY 0.00000 YZ -0.00000 ZZ 0.00000
Dipole Oscillator Strength 0.0329382748
Electric Quadrupole 0.0000001929
Magnetic Dipole 0.0000000133
Total Oscillator Strength 0.0329384810

Occ. 1 a --- Virt. 8 a -0.98992
Occ. 1 a --- Virt. 16 a -0.12916

Root 2 singlet a 19.467775490 a.u. 529.7454 eV

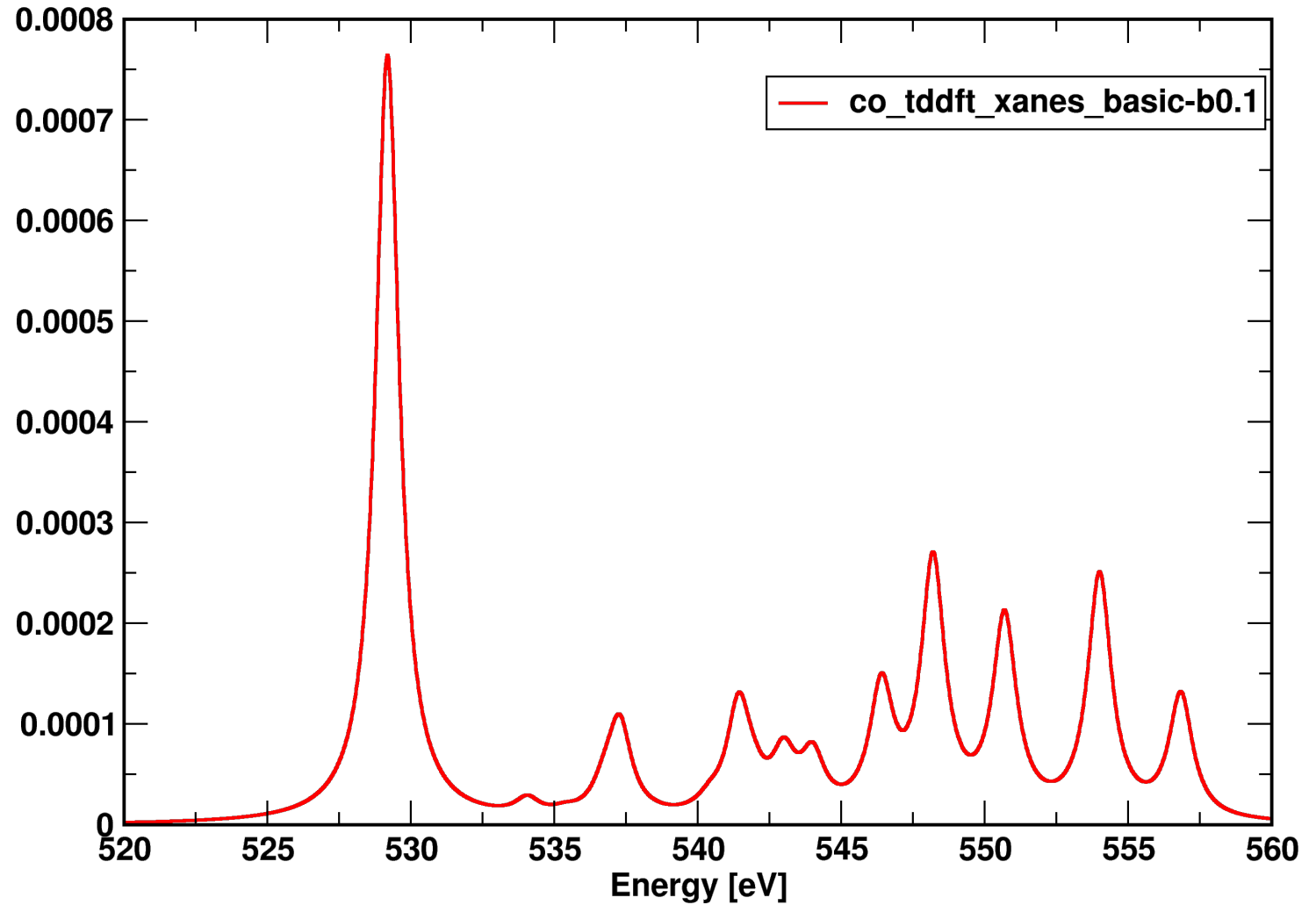
Transition Moments X -0.00001 Y -0.05038 Z -0.00000
Transition Moments XX -0.00000 XY -0.00000 XZ 0.00000
Transition Moments YY -0.00000 YZ 0.00222 ZZ 0.00000
Dipole Oscillator Strength 0.0329382748
Electric Quadrupole 0.0000001929
Magnetic Dipole 0.0000000133
Total Oscillator Strength 0.0329384810

Occ. 1 a --- Virt. 9 a -0.98992
Occ. 1 a --- Virt. 17 a -0.12916

Root 3 singlet a 19.835495901 a.u. 539.7515 eV

...

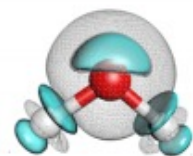
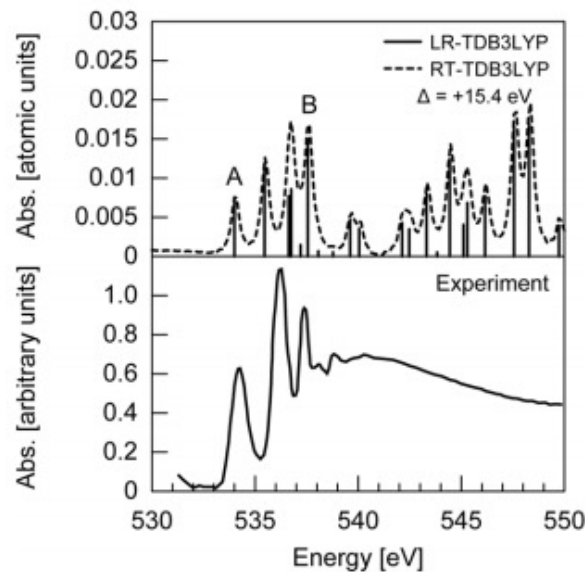
Pre- & Near-Edge X-ray Spectrum



```
python3 nw_spectrum.py -b0.1 -p5000  
<co_tddft_xanes_basic.out>co_tddft_xanes_basic-b0.1
```

Other Examples: O K-edge in Water and O, C K-edges in CO

Water O K-edge gas phase x-ray absorption

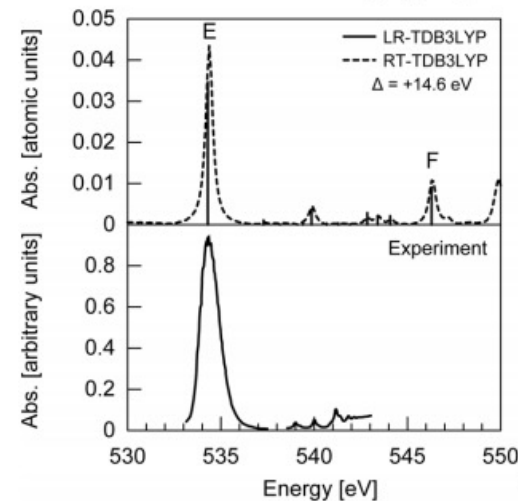


Peak A: 534 eV, \uparrow
O 1s \rightarrow O s, H s, O p_z

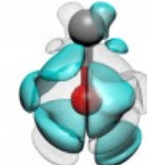


Peak B: 538 eV, \leftrightarrow
O 1s \rightarrow O p_x, H s, H p_x

Carbon monoxide O K-edge gas phase x-ray absorption

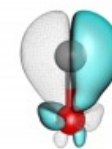
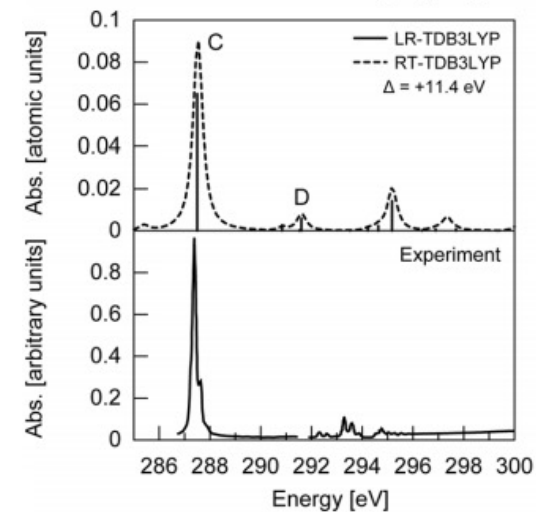


Peak E: 534 eV, \leftrightarrow
O 1s \rightarrow O p_x, C p_x

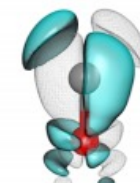


Peak F: 546 eV, \leftrightarrow
O 1s \rightarrow O p_x, C p_x, O d, O f

Carbon monoxide C K-edge gas phase x-ray absorption



Peak C: 287 eV, \leftrightarrow
C 1s \rightarrow C p_x, O p_x

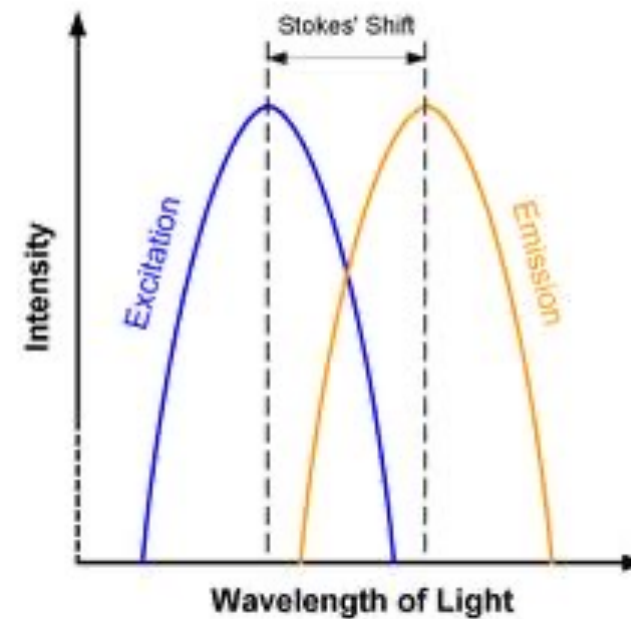
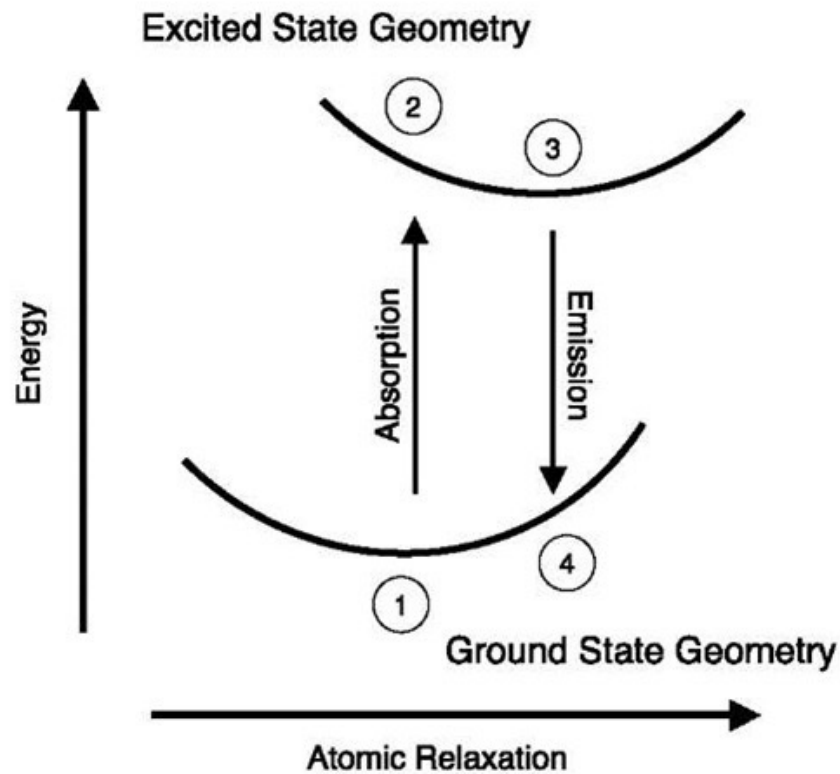


Peak D: 292 eV, \leftrightarrow
C 1s \rightarrow C f, C p_x, O p_x

Details: JCTC, 8, 3284 (2012)

Excited-State Optimization

Optimization on the excited state



Available functionals for TDDFT gradients/optimization/dynamics

Table 1. Definitions of the XC Functionals^a

functional	definition
LDA	Slater + VWN 5
BP86	Becke 88 + Perdew 86
PBE	PBEx + PBEc
BLYP	Becke 88 + LYP
B3LYP	Slater (80%) + Becke 88 (nonlocal, 72%) + HF Exchange (20%) + LYP (81%) + VWN 1 RPA (19%)
PBE0	PBEx (75%) + HF Exchange (25%) + PBEc
BHLYP	Slater (50%) + Becke 88 (nonlocal, 50%) + HF Exchange (50%) + LYP
CAM-B3LYP	CAM-Becke 88 ($\alpha = 0.19, \beta = 0.46, \omega = 0.33$) + LYP
LC-PBE	CAM-PBE ($\alpha = 0.00, \beta = 1.00, \omega = 0.30$) + PBEc
LC-PBE0	CAM-PBE ($\alpha = 0.25, \beta = 0.75, \omega = 0.30$) + PBEc
BNL	Baer-Neuhauser-Livshits ($\alpha = 0.00, \beta = 1.00, \omega = 0.33$) + LYP
LC- ω PBE	LC- ω PBE ($\alpha = 0.00, \beta = 1.00, \omega = 0.30$) + PBEc
LC- ω PBEh	LC- ω PBE ($\alpha = 0.20, \beta = 0.80, \omega = 0.20$) + PBEc
LC-BLYP	CAM-Becke 88 ($\alpha = 0.00, \beta = 1.00, \omega = 0.33$) + LYP

Details: J. Chem. Theory Comput., **2013**, 9 (12), pp 5490–5503

Analytic
TDDFT
Gradients/Opt
in NWChem

Basic TDDFT Optimization Input

```
geometry units angstrom noautosym nocenter
O 0      0      0
C 0      0.00    1.15
end
basis
  * library 6-31G*
end
dft
  xc pbe0
end
task dft optimize
```

```
tddft
  nroots 2
  notriplet
  civecs
  grad ←
    root 1
  end
end
task tddft optimize
```

Defines root on which the excited state
gradient/optimization is performed

Casida Formulation

Cannot be used to describe
excitations in intense fields

Perturbed density \rightarrow first-order correction

Linear response approach \rightarrow frequency domain

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$1 = (X|X) - (Y|Y)$$

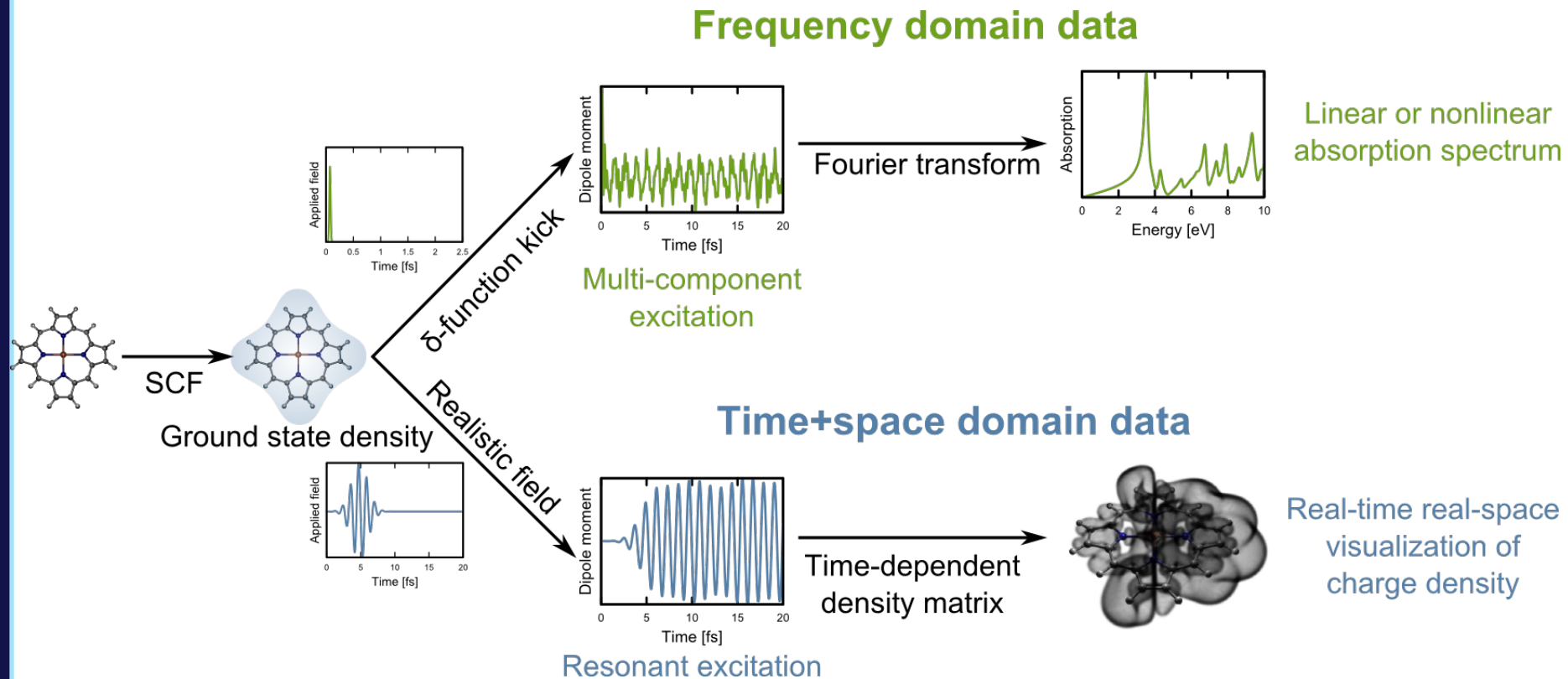
$$A_{ia,jb} = \delta_{ij} \delta_{ab} (\varepsilon_a - \varepsilon_i) + (ia|F_H + F_{xc}|jb)$$

$$B_{ia,jb} = (ia|F_H + F_{xc}|jb)$$

$$F_{xc}(r_1, r_2) = \frac{\partial^2 f}{\partial \rho(r_1) \partial \rho(r_2)}$$

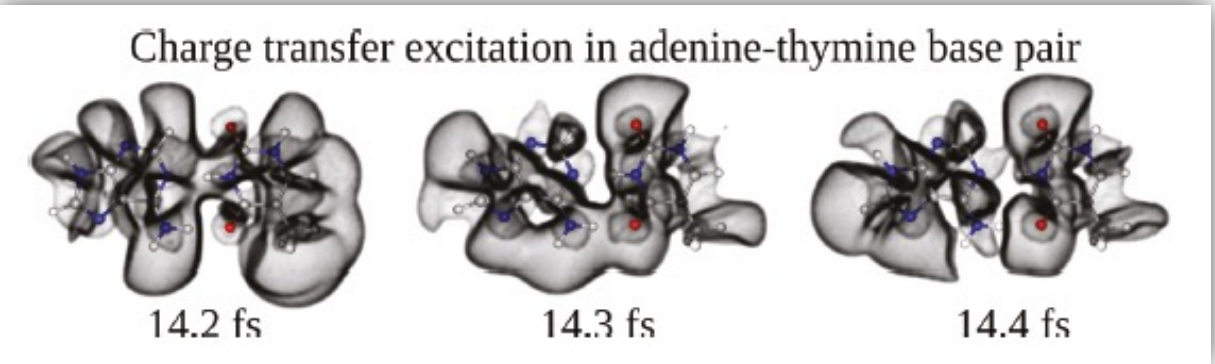
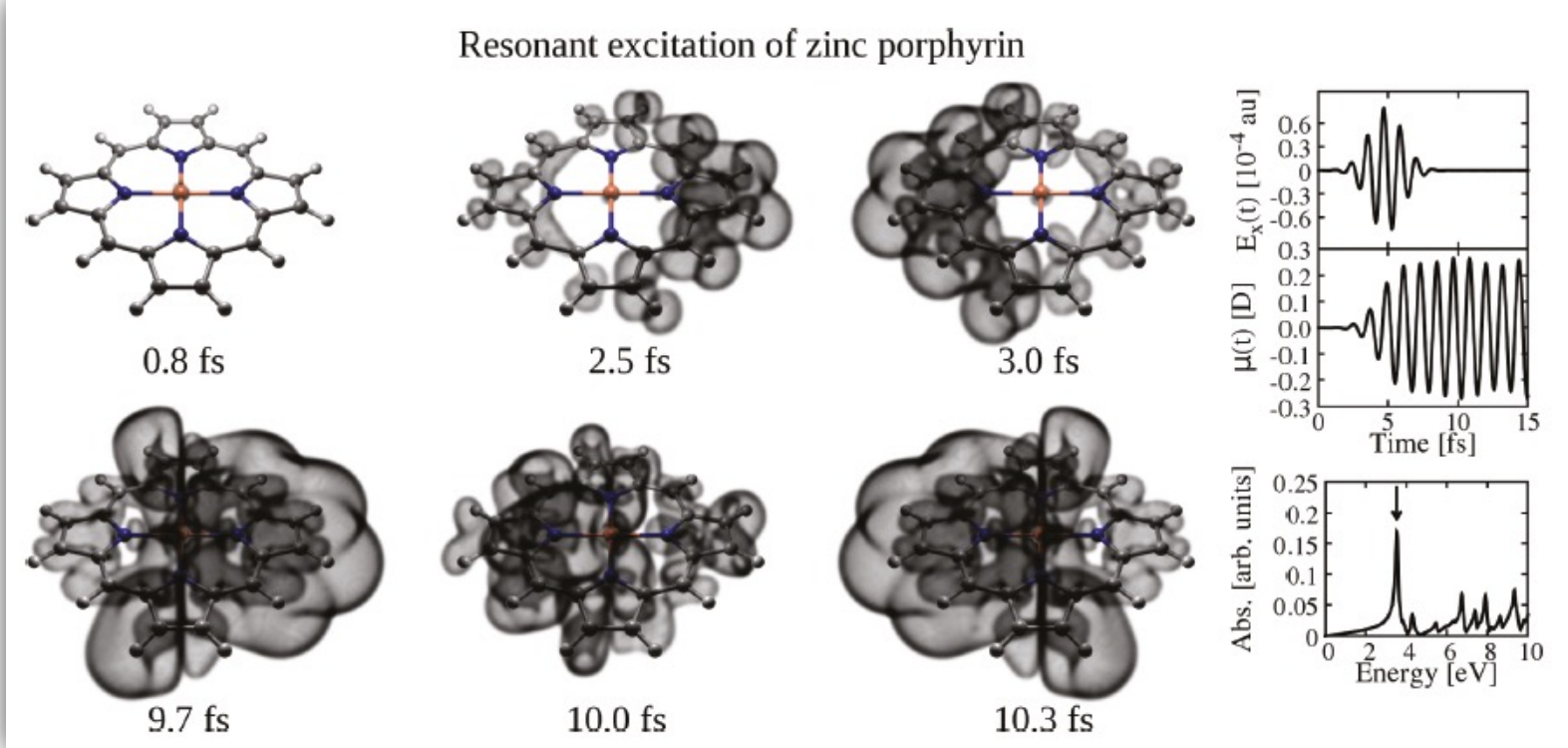
Time-Dependent Density Functional Theory,
Marques et al, Springer 2006

Real-Time TDDFT



- Response beyond perturbation limit, anharmonic effects
- Depends only on the XC potential
- Real-time, real-space \rightarrow full dynamical information
- Insight into ultrafast and nonlinear processes

Ultrafast
Electron
Dynamics



Questions?



Thank you

